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USING A COLLECTIVE OF AGENTS FOR EXPLORATION OF UNDIRECTED GRAPHS

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Abstract. *This paper considers the problem of exploration of finite undirected graphs by a collective of agents. Two agents-researchers simultaneously traverse a graph, read and change labels of graph elements, and send necessary information to the agent-experimenter constructing a representation of the graph being explored. An exploration algorithm is proposed with a linear (with respect to the number of vertices) time complexity and a quadratic space complexity. An optimization procedure is developed that partitions a graph with a view to exploring its parts by different agents. Each of the agents traversing a graph needs two different colors (three colors are used in the aggregate) for exploring a graph. The algorithm is based on the depth-first traversal method.*

Keywords: *exploration of graphs, collective of agents, graph traversal.*

INTRODUCTION

At present, there are many different environments requiring a thorough investigation [1]. This is one of the causes for the active development of the direction called the theory of discrete dynamic systems in mathematical cybernetics. An environment to be investigated is a peculiar kind of a discrete system represented as a model of interaction between a control system and a controlled system. This interaction is often represented as a process of moving a control automaton along the graph of the system being controlled [2]. It is this interpretation that has led to the extensively and intensively developed investigation of the behavior of automata in labyrinths [3–5]. Many publications are devoted to the investigation of a graph with the help of one agent but, at the same time, the exploration of a graph by several agents traversing it is little-investigated. Therefore, the problem of carrying out a systematic investigation of experiments on the exploration of a graph by several agents traversing it is topical. In this exploration, the main problem is the efficiency of interaction of agents with a view to decreasing time and memory expenditures for the exploration. It is necessary to develop traversal algorithms such that, in traversing a graph, the agents do not interfere with each other in work and do not duplicate it.

This article considers a collective consisting of the following three agents: two agents-researchers (ARs) traverse a graph, recolor its elements, and transmit the obtained information to the so-called agent-experimenter (AE). A procedure is developed that allows the agents-researchers to search for new territories for exploration after the completion of exploration of their territories. This procedure solves the problem of idle time for an agent in the case when the initial locations of the agents does not allow them to explore equal parts of a graph and one of the agents has to be idle until the other agent completes the exploration of the remaining part that can considerably exceed the part explored by the idle agent. Therefore, the initial position of the agents exerts appreciable influence on the final time complexity of an algorithm and, in some cases, results in the exploration of the whole graph by one agent (except for the vertex at which the other agent is located).

The agents-researchers interact as a result of repainting graph elements. The algorithms developed earlier [6, 7] for the solution of the problem being considered have cubic (with respect to the number of graph vertices) and quadratic time complexities, respectively, but quadratic space and communication complexities remain unchanged. Let us consider an algorithm for solving this problem in the case when two agents-researchers A and B simultaneously traverse an unknown

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finite nondirected graph without loops and multiple edges and exchange necessary data with the agent-experimenter that reconstructs the graph being investigated. An algorithm is proposed for constructing the AR routes from the graph that allow the AE to exactly reconstruct the graph of the environment being explored. To operate, each AR needs two colors, namely, r and b for A and y and b for B. The algorithm is based on the depth-first traversal method [8], has a linear time complexity, a quadratic space complexity, and the communication complexity equal to $O(n^2 \cdot \log(n))$. In describing the algorithm, results and notations from [6, 7] are used.

BASIC DEFINITIONS AND NOTATIONS

Let $G = (V, E)$ be a connected undirected finite graph without loops and multiple edges, where V is the set of its vertices and E is the set of its edges (two-element subsets (u, v), where $u, v \in V$). (To indicate that some sets V and E are attributes of some graph G, the designations V_G and E_G are used.) We call the triple $((u, v), v)$ the contact point or the contact of an edge (u, v) and a vertex v. We denote by I the set of all contacts in a graph. We call the set $L = V \cup E \cup I$ the set of elements of the graph G. We call a surjective mapping $\mu: L \to \{w, r, y, ry, b\}$, where w, r, y, ry, and b are interpreted as the white, red, yellow, red-yellow, and black colors, the color function of the graph G. The pair (G, μ) is called a colored graph. A sequence $u_1, u_2, \ldots, u_k$ of pairwise adjacent vertices of the graph G is called a path of length k. The vicinity $Q(v)$ of a vertex v is understood to be the set of the graph elements consisting of the vertex v, all the vertices u adjacent to v, all the edges (v, u), and all the contacts $((v, u), v), ((v, u), u)$. The cardinalities of the sets of vertices V and edges E is denoted by n and m, respectively. It is obvious that $m \le n(n-1)/2$. The graph G is reconstructed from the numeration created by the ARs by constructing a graph H isomorphic to G. By an isomorphism of the graph G and the graph H we understand a bijection $\varphi: V_G \to V_H$ such that $(v, u) \in E_G$ if and only if $(\varphi(v), \varphi(u)) \in E_H$. Thus, isomorphic graphs are equal up to vertex names and colorings of their elements. The collective of agents exploring a graph G unknown to them consists of the following three agents: two ARs (A and B) and one AE. The agents-researchers traverse the graph and can change the color of graph elements. Based on AR messages, the AE creates a representation of the graph. The mobile agents A and B have a finite memory increasing at every step. At the beginning of operation, the agents-researchers A and B are placed at arbitrary noncoincident vertices of the graph G and they number these vertices and send their numbers to the AE that places them among vertices of the set V_H. The agents traverse the graph from a vertex v to a vertex u along the edge (v, u) and, during traversing, can change the colors of the vertices v and u, edges (v, u), and contacts $((v, u), v)$, $((v, u), u)$ and also can write numbers in vertices. An AR situated at the vertex v read labels of all elements of the vicinity $Q(v)$ and numbers of its adjacent vertices. Based on this information, the AR determines an edge for the next traversal and some procedure for painting graph elements. The agent-experimenter can send messages to the ARs and can also receive and identify AR messages. It possesses a finite and indefinitely increasing internal memory in which the result of AR functioning at every step is fixed and a representation of the graph G is constructed (which is initially unknown to the agents) in the form of lists of edges and vertices. Note that the ARs explicitly communicate only with the AE and communicate with each other through the AE.

ALGORITHM OF OPERATION OF THE AGENTS-RESEARCHERS

At the beginning of the execution of the algorithm, all graph elements are painted white. In executing the algorithm, graph elements can be painted by the agents-researchers. For short, we call graph elements white, red, yellow, etc. according to their colors.

Let us consider the operating modes of the ARs. In describing these modes, the messages that the ARs send to the agent-experimenter under definite conditions ("MESSAGE_OF_AR_A;" "MESSAGE_OF_AR_B") are parenthesized. The AE, in turn, processes the obtained message and sends data necessary for the completion of a traversal to the corresponding AR.

Ordinary operating mode (OOM). An AR checks the vicinity of the vertex at which it is situated for the presence of white vertices, selects an arbitrary white vertex, and transits to it ("FORWARD_A($x_1, x_2, \ldots, x_q$);" "FORWARD_B($k_1, k_2, \ldots, k_g$)," where x_i (k_i) is the number of the vertex from which the transition is carried out and q (g) is the number of white vertices remained in the vicinity when the agent exits it). Thus, the AR traverses along white vertices and paints the vertices connecting their edges and distant contacts in "its own" color, transits to new vertices, and writes the numbers obtained from the AE in them.

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